Inhomogeneous States of Nonequilibrium Superconductors: Quasiparticle Bags and Antiphase Domain Walls

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Nonequilibrium properties of short-coherence-length s-wave superconductors are analyzed in the presence of extrinsic and intrinsic inhomogeneities. In general, the lowest-energy configurations of quasiparticle excitations are topological textures where quasiparticles segregate into antiphase domain walls between superconducting regions whose order-parameter phases differ by π. Antiphase domain walls can be probed by various experimental techniques, for example, by optical absorption and NMR. At zero temperature, quasiparticles seldom appear as self-trapped bag states. However, for low concentrations of quasiparticles, they may be stabilized in superconductors by extrinsic defects.

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I. INTRODUCTION

In general, studies of superconductors emphasize their equilibrium properties as probed by linear response. Equally important are conditions where the superconductor is driven far from equilibrium. A nonequilibrium state may be achieved, for example, by photoexciting quasiparticles [1,2,3] or injecting them into the superconductor through a tunnel junction [4,5,6]. These experiments have revealed a variety of interesting phenomena, ranging from first-order superconductor-metal transitions, to various instabilities and spatially inhomogeneous states with a laminar structure where either superconducting phases with distinct energy gaps or superconducting and normal phases coexist.

At finite temperature, quasiparticle dynamics in the nonequilibrium state is dominated by scattering with phonons and decay with phonon emission. These processes are characterized by the scattering time τ_s and the lifetime τ_*. While these time scales are long enough compared to $\hbar/\Delta_0$ so that the superconducting energy gap Δ_0 is sharply defined [7], they are also typically of the same order of magnitude, $\tau_s \sim \tau_*$ [8]. As a consequence, it is not obvious that the quasiparticles will equilibrate to a meta-stable state even under steady-state conditions. In contrast, when the lifetime of quasiparticles is the longest time scale, quasiparticles will reach such a state making it possible for new phenomena to emerge. A meta-stable state is obtained, if the system has a symmetry group which makes it possible for the excited and ground states to transform according to different irreducible representations so that the quasiparticle excitations cannot decay. It is clear that in the absence of spin-orbit coupling the only practically meaningful symmetry group is spin-rotational symmetry, because in the superconducting phase broken gauge symmetry destroys the charge conservation. Quasiparticle excitations whose

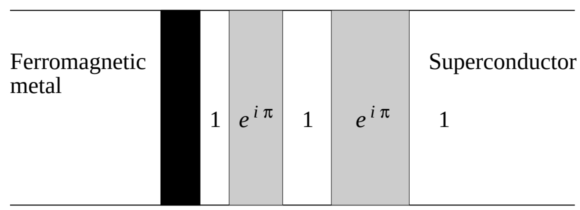

FIG. 1. Schematic description of a tunnel-junction experiment where spin-polarized quasiparticles are injected from a ferromagnetic metal through an insulating barrier to a superconductor. In the superconductor, injected quasiparticles have formed a laminar structure of intervening superconducting domains separated by antiphase domain walls between regions where the neighboring order-parameter phases are shifted by π.

spins are aligned along the same direction may be obtained by using a ferromagnetic metal as a source [9].

In this Note, we examine various meta-stable configurations of quasiparticles and their signatures that might develop when spin-polarized quasiparticles are excited in s-wave superconductors. Our most important finding is that superconductors are unstable against a formation of antiphase domain walls into which the quasiparticles localize and that the local structure and nonuniform spin density makes these topological textures accessible to various experimental probes. In particular, they produce a distinctive optical absorption spectrum that may serve as a unique signature of their presence. In addition, any probe that is sensitive to a local magnetization would lend further support. For example, NMR and μSR might be suitable for this purpose. Figure 1 illustrates schematically a tunnel-junction experiment for generating and detecting antiphase domain-wall textures. Similar experimental construction has been suggested to demonstrate that spin and charge are transported by

separate quasiparticle excitations in a superconductor [10]. We also consider quasiparticle bags which are non-topological states of quasiparticles associated with a local suppression of the order parameter that may appear in the presence of defects when the quasiparticle density is small enough.

Our work is partially motivated by the fact that only few studies exist on self-trapped quasiparticle states in superconductors [11,12] and that either quasiparticle-bag or antiphase domain-wall excitations are usually considered as a curiosity and often disregarded as unphysical [12,13]. Our purpose is to address the question of their existence in mean-field approximation and to examine possible experimental implications by focusing on quasi-one and two-dimensional superconductors which are realized in wires and films. Analogous questions have been studied in the context of antiferromagnets, although no detailed predictions regarding superconductors have been made [13,14].

II. FORMALISM

Our starting point in describing quasiparticle excitations in an s-wave superconductor is the lattice formulation of electrons hopping between nearest-neighbor sites and interacting via an effective two-particle interaction,

$$H = -\tfrac{1}{4}W \sum_{\langle \mathbf{R}\mathbf{r}\rangle\sigma} \psi^\dagger_\sigma(\mathbf{R}+\mathbf{r})\psi_\sigma(\mathbf{R}) - \mu \sum_{\mathbf{R}\sigma} n_\sigma(\mathbf{R}) + U\sum_{\mathbf{R}} n_\uparrow(\mathbf{R})n_\downarrow(\mathbf{R}). \quad (1)$$

Here, $\psi_\sigma(\mathbf{r})$ is the electron operator with spin σ, $\langle\mathbf{R}\mathbf{r}\rangle$ denotes nearest-neighbor sites separated by $\mathbf{r}$, W is the half bandwidth (on a square lattice), μ is the chemical potential, and the operator $n_\sigma(\mathbf{r}) = \psi^\dagger_\sigma(\mathbf{r})\psi_\sigma(\mathbf{r})$ is the conduction electron number density for spin σ. The strength of the pairing interaction U (< 0) is assumed to be intermediate so that the mean-field approximation gives a qualitatively reliable description of the superconducting ground state and the low-energy excitations. Specifically, consider a two-dimensional lattice model where electrons can interact with randomly distributed defects. These defects can either have a magnetic moment or be non-magnetic. The model is defined by the effective Hamiltonian $H_{\rm eff} = H_0 + H_{\rm imp}$, where H_0 describes a BCS superconductor [15] and $H_{\rm imp}$ is the contribution due to impurities. In the mean-field approximation,

$$H_0 = -\tfrac{1}{4}W \sum_{\langle \mathbf{R}\mathbf{r}\rangle} \Psi^\dagger(\mathbf{R}+\mathbf{r})\hat\tau_3\Psi(\mathbf{R}) - \mu\sum_{\mathbf{R}}\Psi^\dagger(\mathbf{R})\hat\tau_3\Psi(\mathbf{R}) - \sum_{\mathbf{R}\mathbf{r}}\Delta(\mathbf{R})\Psi^\dagger(\mathbf{R})\hat\tau_1\Psi(\mathbf{R}), \quad (2)$$

where $\Delta(\mathbf{R})$ is the superconducting gap function and assuming that the pairing of electrons occurs in the spin-singlet channel. The operator $\Psi(\mathbf{r}) = [\psi_\uparrow(\mathbf{r})\ \psi^\dagger_\downarrow(\mathbf{r})]^T$ is a two-component Gor'kov-Nambu spinor, $\hat\tau_\alpha$ $(\alpha = 1,2,3)$ are the Pauli matrices for particle-hole degrees of freedom, and $\hat\tau_0$ is the unit matrix. In a translationally invariant system, the BCS Hamiltonian reduces to

$$H_0 = \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}}(\epsilon_{\mathbf{k}}\hat\tau_3 - \Delta_0\hat\tau_1)\Psi_{\mathbf{k}}, \quad (3)$$

where $\Delta_0 = \Delta(\mathbf{R})$ and $\Psi_{\mathbf{k}} = (\psi_{\mathbf{k}\uparrow}\ \psi^\dagger_{-\mathbf{k}\downarrow})^T$. The fermion operators in real and momentum spaces are related by the unitary transformation, $\psi_\sigma(\mathbf{r}) = N^{-1/2}\sum_{\mathbf{k}}\psi_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}}$, where N is the number of sites in the system. For a square lattice with the nearest-neighbor hopping, the single-particle energy relative to the chemical potential in the normal state is $\epsilon_{\mathbf{k}} = -\frac{1}{2}W(\cos k_x a + \cos k_y a) - \mu$; a is the lattice spacing. In a uniform s-wave superconductor, the energy spectrum of bare quasiparticle excitations is $E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + \Delta_0^2}$. Allowing the excited quasiparticles to relax, the energy spectrum and the order parameter must be modified, as will be discussed below.

The interaction between the conduction electrons and the impurities in the superconductor is given by the Hamiltonian

$$H_{\rm imp} = \sum_{\mathbf{r}}[V(\mathbf{r})n(\mathbf{r}) + J\mathbf{S}(\mathbf{r})\cdot\mathbf{s}(\mathbf{r})], \quad (4)$$

where $n(\mathbf{r}) = \sum_\sigma n_\sigma(\mathbf{r})$ and $\mathbf{s}(\mathbf{r}) = \frac{1}{2}\sum_{\sigma\nu}\psi^\dagger_\sigma(\mathbf{r})\hat\tau_{\sigma\nu}\psi_\nu(\mathbf{r})$ are the conduction electron number density and spin density operators. In the case of point like impurities located at sites $\mathbf{r}_n$, the potential (scalar) and magnetic scattering terms have the forms $V(\mathbf{r}) = \sum_n V_n\delta_{\mathbf{r}\mathbf{r}_n}$ and $\mathbf{S}(\mathbf{r}) = \sum_n \mathbf{S}_n\delta_{\mathbf{r}\mathbf{r}_n}$. Typically, the distribution of impurities is assumed to be random whereas the magnitude of scalar and magnetic scattering are constant, $V_n = V$ and $w = JS/2$, where $S = |\mathbf{S}_n|$. For later emphasis, it is useful to introduce here a particle-hole transformation generated by the operator $\hat\tau_1$:

$$\Psi(\mathbf{r}) \to \Psi'(\mathbf{r}) = (-1)^{\mathbf{r}}\hat\tau_1\Psi(\mathbf{r}). \quad (5)$$

At half filling $(\mu = 0)$, the BCS Hamiltonian H_0 on a square lattice is invariant under this transformation. Moreover, if the impurity moments are aligned along the same direction and there is no potential scattering, the impurity Hamiltonian $H_{\rm imp}$ will also be invariant under the same transformation. Potential scattering and randomly oriented impurity moments break particle-hole symmetry of this nature.

Given that the pairing of electrons occurs in the spin-singlet state, the superconducting order parameter (amplitude) can be expressed in the form

$$F(\mathbf{R},\mathbf{r}) = \tfrac{1}{2}\sum_{\sigma\nu}(i\hat\tau_2)_{\sigma\nu}\langle\psi_\nu(\mathbf{R}+\mathbf{r})\psi_\sigma(\mathbf{R})\rangle. \quad (6)$$

The relation between the order parameter and the gap function is given by the equation

$$\Delta(\mathbf{R}) = -UF(\mathbf{R}, \mathbf{r} = 0). \quad (7)$$

The on-site pairing interaction U is assumed to be instantaneous in time. Thus, the energy cutoff in the gap equation is set by the bandwidth. In our numerical approach, the gap equation is solved self-consistently with a given number of quasiparticle excitations on finite size lattices with periodic boundary conditions. In our numerical examples, the strength of the interaction U is chosen so that in the absence of impurities and quasiparticle excitations the energy gap is $\Delta_0/W = 0.1$.

III. MAPPING TO AN ANTIFERROMAGNET

The Hamiltonian (1) can be transformed to a model where the on-site interaction is repulsive. In the case of longer-range interactions, Ising-like terms are generated. On a bipartite lattice, this is achieved by a particle-hole transformation on the down spins,

$$\begin{aligned} &\psi_\uparrow(\mathbf{r}) \to \psi_\uparrow(\mathbf{r}), \\ &\psi_\downarrow(\mathbf{r}) \to (-1)^{\mathbf{r}} \psi_\downarrow^\dagger(\mathbf{r}). \end{aligned}$$

In this transformation, the particle number operator transforms to the z-component of the spin density operator: $n(\mathbf{r}) \to 2s_z(\mathbf{r}) + 1$, and vice versa. The Hamiltonian is mapped into

$$\begin{aligned} H = &-\tfrac{1}{4}W \sum_{\langle \mathbf{Rr} \rangle \sigma} \psi_\sigma^\dagger(\mathbf{R}+\mathbf{r})\psi_\sigma(\mathbf{R}) + h_z \sum_{\mathbf{r}} s_z(\mathbf{r}) \\ &+ 2U \sum_{\mathbf{r}} s_z(\mathbf{r}) s_z(\mathbf{r}), \end{aligned} \quad (8)$$

where $h_z = U - 2\mu$ is an effective magnetic field along the z-axis. Thus, for the Hubbard model, the particle-hole transformation changes the sign of the on-site interaction U.

The superconductor has U(1) symmetry associated with the phase of the order parameter. Because the real and imaginary parts of the order parameter are transformed to the x and y components of the spin, a gauge transformation corresponds a rotation of the spin in the xy plane.

The particle-hole transformation establishes one-to-one correspondence between the ground states of the attractive and repulsive Hubbard models. For example, consider the attractive Hubbard model away from half filling so that the average electron density $\langle n \rangle < 1$ and the average spin density $\langle s_z \rangle = 0$. The particle-hole transformation maps it into the half-filled, repulsive Hubbard model with the effective magnetic field h_z. Its ground state has a transverse antiferromagnetic order because in this way the system can lower its energy by generating a small ferromagnetic component parallel to the z-axis. Therefore, in the ground state, $\langle n \rangle = 1$ and $\langle s_z \rangle < 0$. Reversing the transformation, the transverse antiferromagnetic order parameter is mapped to a superconducting order parameter in the attractive Hubbard model.

Next, consider the attractive Hubbard model away from half filling but now in the magnetic field so that $\langle n \rangle < 1$ and $\langle s_z \rangle < 0$. This model is mapped by the particle-hole transformation to the repulsive Hubbard model away from half filling. It has a ground state which is described by antiphase domain walls between antiferromagnetically ordered spins [13]. By virtue of the particle-hole transformation, it is clear then that the attractive Hubbard model with spin-polarized quasiparticles has a superconducting ground state where the superconducting domains with the opposite signs of the order parameter are separated by antiphase domain walls into which the excess spin is localized.

IV. THE CONTINUUM MODEL

Although we are mostly interested in quasi-two-dimensional superconductors, it is useful to consider one-dimensional systems where many ideas can be examined analytically. Indeed, for quasi-one-dimensional systems, a fruitful connection between the BCS and SSH Hamiltonians can be made. The latter one describes, for example, conducting polymers where the order parameter $\Delta(x)$ represents the lattice distortion [16]. In our case, such systems can be organic superconductors or wires whose thickness is smaller than the coherence length ξ_0. In the weak-coupling limit, additional progress is achieved by considering a continuum field theory, which can be derived because the coherence length is much longer than the lattice spacing, $\xi_0 \gg a$.

Since we are interested in low-energy and long-wavelength phenomena, the electronic degrees of freedom can be expressed by slowly varying fields $\psi_{\pm\sigma}(x)$ describing the left $(+)$ and right $(-)$ moving electrons,

$$\psi_\sigma(x)/\sqrt{a} = \psi_{+\sigma}(x) e^{ik_F x} + \psi_{-\sigma}(x) e^{-ik_F x}, \quad (9)$$

where k_F is the Fermi wave vector. Defining the four-component spinor as $\Psi(x) = [\Phi_\uparrow(x) \ \Phi_\downarrow^*(x)]^T$, where $\Phi_\sigma(x) = [\psi_{+\sigma}(x) \ \psi_{-\bar{\sigma}}^\dagger(x)]^T$, the BCS Hamiltonian (2) becomes

$$H_0 = \int dx\, \Psi^\dagger(x)[v_F \hat{p} - \Delta(x)\hat{\tau}_1]\Psi(x). \quad (10)$$

The momentum operator is $\hat{p} = -i\hbar\hat{\tau}_3\partial_x$, where $v_F = (2ta/\hbar)\sin k_F a$ is the Fermi velocity. Similarly, the gap equation can be rewritten in the form [17]

$$\Delta(x) = -\tfrac{1}{2} aU \langle \Psi^\dagger(x)\hat{\tau}_1\Psi(x) \rangle. \quad (11)$$

These equations are formally equivalent to those of the TLM model [18], which is the continuum limit of the

SSH model. For example, at zero temperature, the superconducting energy gap is $\Delta_0 = 2We^{-1/\lambda}$, where the dimensionless interaction is $\lambda = N_F|U|$; N_F is the density of states at the Fermi energy in the normal state. Similarly, the coherence length is $\xi_0 = \hbar v_F/\Delta_0$.

It is now straightforward to determine the nonequilibrium properties of the quasi-one-dimensional s-wave superconductor. In particular, it is obvious that injecting spin-polarized electrons into the system, they form solitons. They are topological excitations of the system, acting as domain walls between two ground states that differ by the sign of the order parameter Δ. The energy of the soliton is $E_{dw} = 2\Delta_0/\pi$ and the order parameter is $\Delta(x) = \Delta_0 \tanh[(x - x_0)/\xi_0]$, where x_0 is the location of the center of the soliton. At low densities of injected electrons, single quasiparticle bags may appear. They are counterparts of polarons; thus, also their spatial form as well as their energy, $E_{qp} = \sqrt{2}E_{dw}$, is known exactly. While in inhomogeneous superconductors individual quasiparticles may diffuse until they become trapped into defects, they are not generic solutions, because at finite concentration of quasiparticle excitations they "phase separate" forming domain walls.

V. ANTIPHASE DOMAIN WALLS

While at zero temperature quasiparticle bags are not generic excitations of the superconductor, they may appear as long-lived states because of defects. This may happen if they are injected into the system at low rate so that they can migrate without scattering from other quasiparticle excitations long distances before they are trapped to defects. Note that, in addition to magnetic impurities, a local order-parameter suppression caused by nonmagnetic impurities leads to bound states in the superconducting energy gap, albeit their binding energies are necessarily small [19]. Figure 2 illustrates a situation which is obtained when the quasiparticle concentration is small and there are magnetic impurities in the system. For simplicity, the magnetic impurities are assumed to be ferromagnetically ordered producing a maximal trapping potential. The bound quasiparticle states yield two peaks in the density of states,

$$\mathcal{N}(\omega) = -\frac{1}{2\pi}\sum_{\mathbf{r}\sigma} \operatorname{Im} G_{\sigma\sigma}(\mathbf{r},\mathbf{r};\omega + i0^+), \qquad (12)$$

in the energy gap. The oscillations in $\mathcal{N}(\omega)$ for $|\omega| > \Delta_0$ are due to the finite size effects.

With an increasing quasiparticle concentration, individual quasiparticle excitations become unstable towards a spontaneous formation of antiphase domain walls. This tendency is depicted in Fig. 3, where the number of quasiparticles is not large enough to form a domain wall that would extend all the way through the system. Instead,

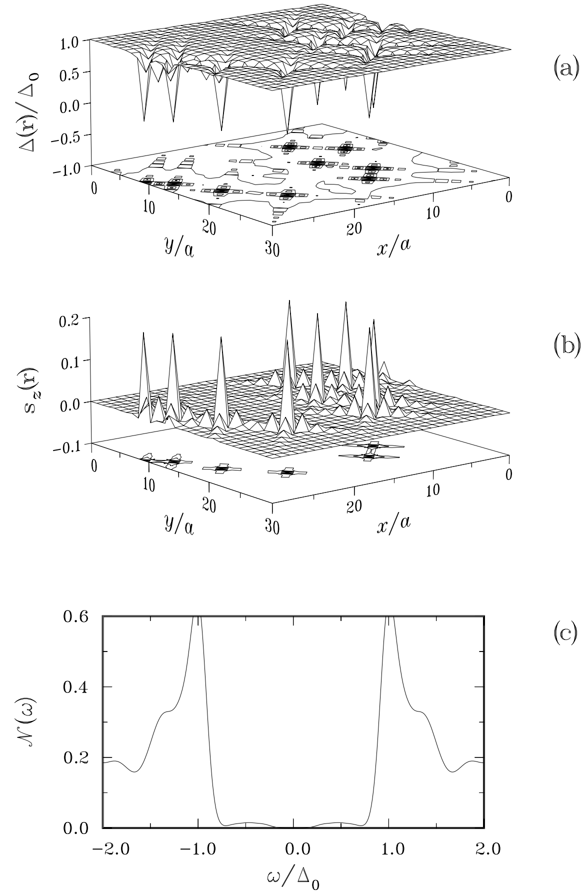

FIG. 2. (a) The energy gap, (b) the spin density, and (c) the density of states of a localized solution of spin-polarized quasiparticles injected into an s-wave superconductor with magnetic impurities. This configuration of well separated quasiparticle bags is obtained self-consistently on a square lattice with the lattice spacing a, $\Delta_0/W = 0.1$, $\pi N_F w = 0.3$, and $\mu = 0$. The concentration of quasiparticles and magnetic impurities equals 1%.

a closed domain-wall loop is formed. Because the order parameter changes sign across the domain wall, there are midgap states. The finite length of the domain wall leads to the level repulsion yielding the density of states that has a minimum at zero energy. As the system is half filled and either there are no impurities or their moments are parallel to each other, the effective Hamiltonian $H_{\rm eff}$ is invariant under the particle-hole transformation, Eq. (5). Consequently, the density of states, depicted in Figs. 2 and 3, is symmetric relative the zero energy.

For a finite concentration of quasiparticles, it becomes energetically favorable to form domain walls with infinite length; see Fig. 4. This allows all the quasiparticles to occupy the midgap states. Domain walls may become pinned to defects either because the defects have a magnetic moment or because the defects suppress the order

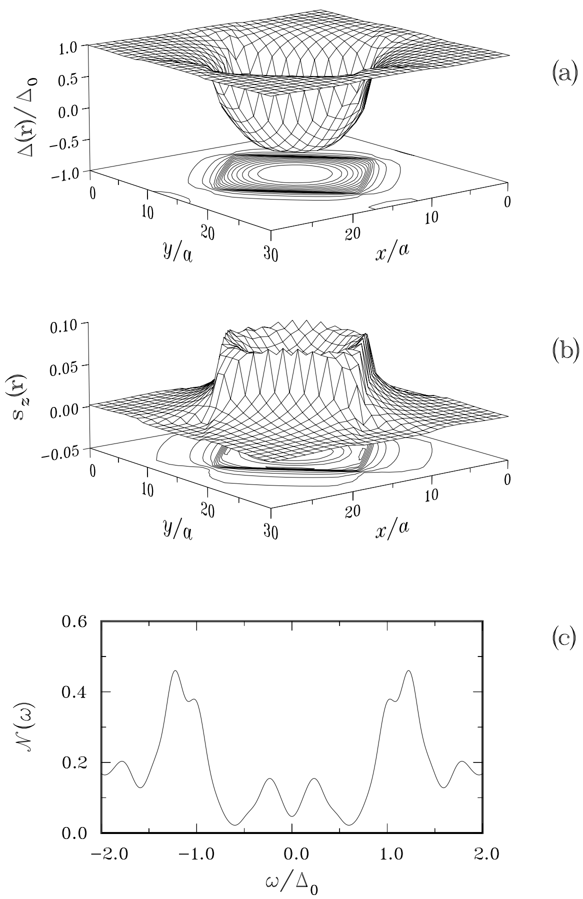

FIG. 3. (a) The energy gap, (b) the spin density, and (c) the density of states when a finite number (40) of spin-polarized quasiparticles is injected into an s-wave superconductor. This configuration is obtained self-consistently on a square lattice with the lattice spacing a, $\Delta_0/W = 0.1$, and $\mu = 0$. No impurities are present.

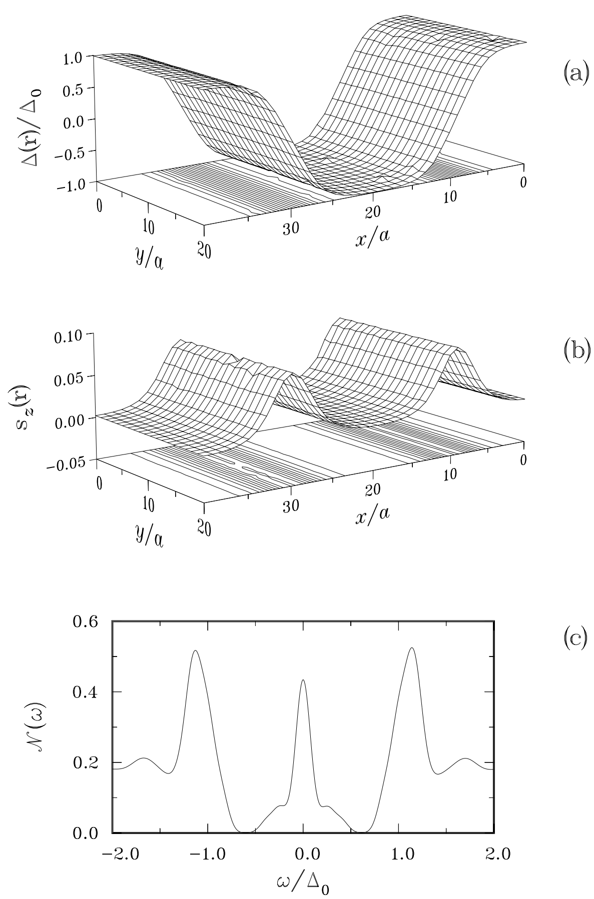

FIG. 4. (a) The energy gap, (b) the spin density, and (c) the density of states in an s-wave superconductor with 5% spin-polarized quasiparticles. The antiphase-domain-wall configuration is obtained self-consistently on a square lattice with the lattice spacing a, $\Delta_0/W = 0.1$, $\mu = 0$, $\pi N_F V = 0.3$, and $n_{\rm imp} = 2\%$.

parameter locally, and this local suppression then pins a domain wall. In the case of extended defects, domain walls may find it preferable to wind through these defects.

All the quasiparticle and domain-wall textures are charge neutral when the system has particle-hole symmetry at the Fermi energy. In this regard, quasiparticle bags can be described as spinons [10], because they carry spin but no charge. On a square lattice with the nearest-neighbor hopping, this happens exactly at half filling ($\mu = 0$). However, if particle-hole symmetry at the Fermi energy is broken, self-consistently determined quasiparticle configurations usually acquire charge, because they are a linear combination of plane-wave states with an energy spread $\Delta\epsilon \sim \hbar v_F/\xi_0$ about the Fermi energy. Similarly, away from half filling, the domain walls become charged, although their total charge per unit length can be quite small. This feature is naturally understood by considering the repulsive Hubbard model with a finite effective magnetic field which induces a small longitudinal ferromagnetic component. In the superconductor, this component is equivalent to a non-zero charge density. In contrast, bare quasiparticle excitations at the Fermi surface ($\mathbf{k} = \mathbf{k}_F$) behave as spinons irrespective of the energy spectrum in the normal state.

Finally, consider the stability of domain-wall solutions against a formation of isolated quasiparticle bags. Their energies per particle can be computed numerically. In two dimensions, the energy of a vertical domain wall per particle is estimated as $E_{dw} \simeq 0.66\Delta_0$ and the energy of a quasiparticle bag as $E_{qp} \simeq 0.86\Delta_0$. These estimates are in agreement with those computed in the antiferromagnetic system for vertical domain walls [13] and spin polarons [20]. Thus, approximately at the temperature $T_* \sim \Delta_0/5$ a considerable fraction of domain walls begins to evaporate forming isolated quasiparticle

bags. It is interesting to compare this temperature with the critical temperature of the superconductor, which is $T_c \sim \Delta_0/2$. Thus, there is a sizable temperature regime below T_c where most of the excitations appear as isolated quasiparticle bags. At low enough temperatures, $T \lesssim T_c/3$, domain-wall textures are thermodynamically favored over non-topological quasiparticle configurations.

VI. OPTICAL ABSORPTION

The optical absorption provides a specific probe to various inhomogeneous states of nonequilibrium superconductors. The optical absorption is the real part of the complex conductivity, $\sigma'_{ab}(\omega) = \mathrm{Re}\,\sigma_{ab}(\omega)$ $(a, b = x, y)$, where

$$\sigma_{ab}(\omega) = -\frac{1}{i\omega}\big[\Lambda_{ab}(\mathbf{q}=0, \omega + i0^+) + \frac{ne^2}{m^*}\delta_{ab}\big]. \quad (13)$$

The ratio between the density of charge carriers and their effective mass is defined as $n/m^* = -a^2\langle H_{\mathrm{kin}}\rangle/2$, where H_{kin} is the kinetic-energy part of the Hamiltonian H. The current-current correlation function is given by the formula

$$\Lambda_{ab}(\mathbf{q}, t) = -\langle T j_a(\mathbf{q}, t) j_b(-\mathbf{q}, 0)\rangle, \quad (14)$$

and its Fourier transform is

$$\Lambda_{ab}(\mathbf{q}, \omega) = \frac{1}{N}\int_0^\infty dt\, e^{i\omega t}\Lambda_{ab}(\mathbf{q}, t). \quad (15)$$

The current operator in Heisenberg picture is defined as $j_a(\mathbf{q}, t) = e^{iHt} j_a(\mathbf{q}) e^{-iHt}$, where

$$j_a(\mathbf{q}) = \tfrac{i}{4} eaW \sum_{\langle \mathbf{Rr}\rangle} \Psi^\dagger(\mathbf{R} + \mathbf{r}_a)\hat{\tau}_0 \Psi(\mathbf{R}) e^{-\mathbf{q}\cdot\mathbf{r}}. \quad (16)$$

It is useful note that optical absorption obeys the sum rule:

$$\int_0^\infty d\omega\, \sigma'_{aa}(\omega) = \frac{\pi}{2}\frac{ne^2}{m^*}. \quad (17)$$

It is a quantity describing any state that is linearly perturbed by the electric field. Typically, it is associated with the equilibrium state. For non-equilibria states, such as the domain walls and quasiparticle bags, the sum rule must be modified.

In the normal state, the optical conductivity has the Drude form

$$\sigma'_{aa}(\omega) = \frac{ne^2\tau}{m^*}\frac{1}{(\tau\omega)^2 + 1}, \quad (18)$$

where τ^{-1} is the scattering rate due to the impurities. In the limit of dilute concentration of impurities, it can be approximated as

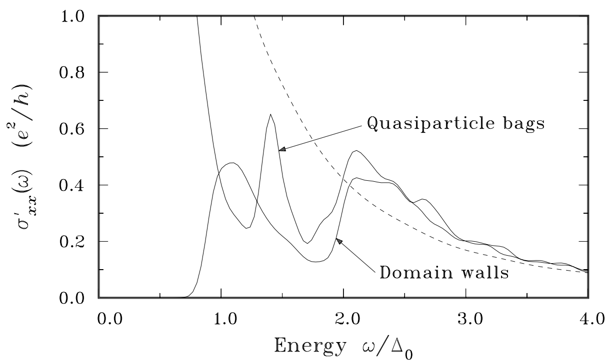

FIG. 5. Optical absorption $\sigma'_{xx}(\omega)$ in an s-wave superconductor in the presence of quasiparticle excitations forming a domain-wall lattice and localized quasiparticle bags. The concentration of quasiparticles in both cases is 1%. These configurations are obtained self-consistently in one dimension for $\Delta_0/W = 0.1$, $\mu = 0$, $\pi N_F V = 0.3$, and $n_{\mathrm{imp}} = 5\%$. The dashed line denotes the optical absorption (Drude-like) obtained in the normal state ($\Delta_0 = 0$).

$$\tau^{-1} = \frac{2n_{\mathrm{imp}}}{\pi N_F}\sin^2\delta, \quad (19)$$

where δ is the phase shift for s-wave scattering and n_{imp} is the impurity concentration. For point-like impurities, the scattering phase shift is obtained from the equation $\cot\delta = c$, where $c = (\pi N_F V)^{-1}$, for nonmagnetic impurities, and $c = (\pi N_F w)^{-1}$, for magnetic impurities. Below, as a reference, the numerically determined optical conductivity in the presence of randomly distributed impurities in the normal state ($\Delta_0 = 0$) is also shown. It is well described [21] by the Drude form, Eq. (18).

In the superconductor, the zero-temperature optical conductivity typically has a threshold of $2\Delta_0$ due to the superconducting energy gap in the electronic spectrum at the Fermi energy. Quasiparticle bags and antiphase domain walls introduce states within this energy gap that can be used as a characteristic signature of them. Figure 5 illustrates the optical absorption when the injected quasiparticles form either isolated bag states pinned to nonmagnetic impurities or domain walls. In the former case, there is a very large peak in the absorption that comes from the excitation processes from the localized bag states to states just below the energy gap. Because the order-parameter suppression occurs on the length scale determined by the coherence length, it acts as an attractive potential with a finite range that can bind states just below the energy gap. Thus, in addition to the state occupied by the quasiparticle, the order-parameter relaxation may admit additional discrete states below the energy gap. Transitions between these states have a very large oscillator strength. One can also see a peak at $2\Delta_0$, which is due to the pair-breaking processes across the energy gap. In the case of domain walls, the optical ab-

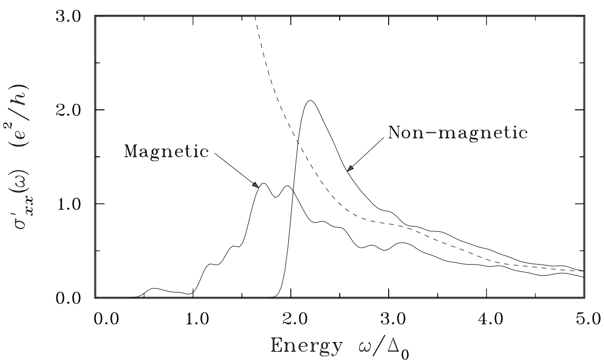

FIG. 6. Optical absorption $\sigma'_{xx}(\omega)$ in an s-wave superconductor with 5% scalar ($\pi N_F V = 0.6$) and magnetic ($\pi N_F w = 0.6$) impurities in the absence of quasiparticle excitations. The ground-state configuration is obtained self-consistently in one dimension for $\Delta_0/W = 0.1$ and $\mu = 0$. The dashed line denotes the optical absorption (Drude-like) obtained in the normal state ($\Delta_0 = 0$).

sorption begins at Δ_0 due to the midgap states. One can therefore clearly distinguish between these two nonequilibrium states based on the optical absorption.

Impurities yield a quite different absorption spectrum in the absence of quasiparticle excitations; see Fig. 6. Nonmagnetic impurities produce a spectrum that has a clear threshold near $2\Delta_0$ and a peak, whereas magnetic impurities yield a relatively smooth absorption profile which may extend deep below $2\Delta_0$, depending on the coupling strength between electrons and impurity moments. For a high enough concentration of magnetic impurities, the superconductor becomes gapless and the absorption will begin at zero energy.

VII. FINAL REMARKS

Based on both analytical and numerical approaches, we have demonstrated that s-wave superconductors driven away from equilibrium exhibit interesting topological textures. They develop as quasiparticles in nodeless superconductors segregate forming antiphase domain walls in the superconducting order parameter and in this manner induce low-energy excitations into which quasiparticles relax. Their inhomogeneous structure has clear experimental implications. For example, a nonuniform spin density associated with domain walls should be accessible to any probe that is sensitive to a spatially varying magnetization. Moreover, optical absorption provides another unambiguous tool for exploring these textures.

We have assumed that the lifetime τ_* of the quasiparticles in the excited state is much longer than the scattering time τ_s so that a metastable state is reached. This will require the use of spin-polarized quasiparticles, which may not always be feasible. A qualitatively similar situation may be created by maintaining a steady state of unpolarized quasiparticles by continuously pumping quasiparticles into excited states. Even though a genuinely metastable state may not develop because $\tau_* \sim \tau_s$, the fact that τ_* can be many orders of magnitude longer than the time scale associated with the superconducting energy gap, $\hbar/\Delta_0$, suggests that some of the features explored here may actually be relevant for such states, too. Time resolved techniques are an ideal tool to probe their properties.

While in gapless superconductors, such as in d-wave superconductors, it is no longer clear that quasiparticle excitations will lead to antiphase domain walls, various external defects that suppress the order parameter may locally favor a phase shift. Such textures may appear in magnetic superconductors with static spin-density-wave ordering where the phases of the magnetic and superconducting order parameters intertwine to form a new collective state with midgap quasiparticle states.

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